

Probability Distribution Modelling of Nigeria's Economic Growth Rate

^{1*}Bodunwa, O. K., ²Adewole, A.I. and ¹Ayodele S.

¹Federal University of Technology, Akure.

²Tai Solarin University of Education, Ijebu-Ode

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ABSTRACT

The probability distribution that best describes the Nigeria economic growth rate from year 1960-2019 was investigated. The secondary data obtained from www.macrotrends.net on Gross Domestic Product (GDP) in other to measure the Economic Growth Rate of Nigeria was used. Six theoretical probability distributions were fitted using Normal distribution, Weibull distribution, Pareto distribution, Cauchy distribution, Gamma distribution, lognormal distribution. The Weibull distribution fitted the data as confirmed by the Kolmogorov Smirnov goodness of fit test and the best fit distribution was selected using the Akaike Information Criteria, Bayes Information Criteria. Probability forecast was done using the best fit probability distribution. This shows that the probability of obtaining an increase in the value of economic growth rate in less than 5 years in Nigeria is 8% which is highly low. Also, the probability of obtaining an increase in the value of Economic Growth rate in less than 10 years in Nigeria is 14% which is also low. Lastly, the probability of obtaining an increase in the value of economic growth rate above 10 years in Nigeria is 85% which is high. Meaning that before we can have a meaningful increment in our economic growth in Nigeria is after ten years of this study

Keywords: Probability distribution, GDP, Akaike Information Criteria, Weibull distribution

Introduction

The Gross Domestic Product (GDP) is one of the key indicators used to measure the well-being of a country's economy. It is also used to govern the standard of living of individuals in an economy. The Gross Domestic Product is used in the comparison of relative growth rate of economies throughout the world. Samuelson (2005), explained Gross Domestic Product as the name given to the total market value of the final goods and services produced within a nation during a given year. From his description of Gross Domestic Product, GDP is used for many purposes but the most important one is to measure the overall

performance of an economy and this overall performance could be measured as a flow of final products or as a flow of cost. Both approaches will yield the same total GDP since profit, is a residual. There are two types of GDP in existence which includes; Real GDP and Nominal GDP (Krugman 2021). The Real GDP is the measurement of economic output of a country minus the effect of inflation and Nominal GDP is the measurement that leaves the price changes in the estimate. A probability distribution is a table or an equation that links each outcome of a statistical experiment with its probability of occurrence (Robert, 2008).

Corresponding Author: Bodunwa, O. K.

Federal University of Technology, Akure

Email: okbodunwa@futa.edu.ng



It is a function that describes the likelihood of obtaining the possible values that a random variable can assume. In other words, the values of the variable vary based on the underlying probability distribution.

Evans *et al.* (2010), stated that probability distribution is a mathematical description of a random phenomenon in terms of its sample space and the probabilities of events (subsets of the sample space).

There are many different classifications of probability distributions. Some of them include the normal distribution, chi square distribution, binomial distribution, and Poisson distribution, Weibull, Pareto, Gamma etc. (Robert, 2008). The different probability distributions serve different purposes and represent different data generation processes.

Therefore, this study focuses on theoretical probability fitting of Nigeria economic growth rate. The theoretical probability distributions that will be considered includes (Normal, Weibull, Pareto, Cauchy, Gamma, log normal). In order to know the probability distribution that best describe the data, Kolmogorov-Smirnov goodness of fit, Akaike information criterion (AIC), and Bayesian Information Criterion (BIC) will be considered for confirmation.

Mahmood and Stockhammer (2014), studied 'Fitting Probability Distribution to Economic Growth: A Maximum Likelihood Approach', the growth rate of the gross domestic product (GDP) usually carries heteroscedasticity, asymmetry and fat-tails. Three important and significantly heteroscedastic GDP series are studied, these were removed and the distributions of the altered series are then compared to Normal (N), Normal Mixture (NM), Normal Asymmetric Laplace (NAL) distributions and a Student's t Asymmetric Laplace (TAL) distribution mixture are proposed for distributional fit comparison of GDP series after removing heteroscedasticity. The maximum likelihood method is used for estimation of the parameters of the distributions. Based on the

results of different accuracy measures, goodness of fit tests and plots, we find that in the case of asymmetric, heteroscedastic and highly leptokurtic data the TAL-distribution fits better than the alternatives. In the case of asymmetric, heteroscedastic but less leptokurtic data the NM fit is superior.

Stockhammer and Oller (2007) studied the Probability Distribution of Economic Growth. They said normality is often mechanically and without sufficient reason assumed in econometric models.

In their findings, three important and significantly heteroscedastic GDP series are studied. Heteroscedasticity is removed and the distributions of the altered series are then compared to Normal, Normal Mixture and Normal Asymmetric Laplace (NAL) distributions. NAL represents a skewed and leptokurtic distribution, which is in line with the Aghion and Howitt (1992) model for economic growth, based on Schumpeter's idea of creative destruction. Statistical properties of the NAL distributions are provided and it is shown that NAL competes well with the alternatives.

In the Schumpeterian world growth which is driven endogenously by investments into Research and Development (R&D), leading to better products, which initially capture monopoly profits. The quality improvements occur randomly over time. The main contributions to endogenous growth are given by Romer (1986) and Lucas (1988).

Therefore, the probability distribution modeling of Nigeria's economic growth was evaluated using the method of Goodness-of-fit. Six probability distributions were used for the modeling

Materials and Methods

In modeling the annual economic growth rate in Nigeria, the theoretical probability distributions (Normal, Weibull, Pareto, Cauchy, Gamma, log normal) were fitted theoretically and then using the best fit, which was determined by the Kolmogorov – Smirnov goodness of fit test and then used to make probability forecast. The two model criteria that

were used to select the best fit probability distribution models are the Akaike information criteria (AIC) and the Bayesian information criteria (BIC). The method of maximum likelihood estimation was used to estimate the parameters of the probability distributions. Descriptive statistics was conducted, which helps to describe or summarize the data set. The data used is a secondary data collected from year 1960-2019, from www.macrotrends.net on Gross Domestic Product (GDP).

The Akaike Information

The Akaike Information criterion (AIC) provides a relative measure of Goodness of Fit among different distributions of interest. This means that using AIC on one single distribution provides no information about the Goodness of fit. AIC measures the fitness by emphasizing on Goodness of fit while deterring over fitting. The distribution that produces the smallest AIC value is the preferred choice. The AIC value is given by (Akaike (1974)

$$AIC=2k-2\ln L$$

Such that k is the number of parameters of the distribution and L is the likelihood function mentioned above for the derived distribution. To compare distributions that has small number of data, the corrected AIC (AICc) is used Hurvich and Tsai (1989)

$$AICc = AIC + \frac{2k(k+1)}{n-k-1}$$

The AIC and AICc however does not measure an absolute Goodness of Fit and hence if none of the chosen distributions is not a good fit, this test does not show that.

Bayesian Information Criteria.

In statistics, the Bayesian information criterion (BIC) or Schwarz criterion (also SBC, SBIC) is a criterion for model selection among a finite set of models. It is based, in part, on the likelihood

function, and it is closely related to Akaike information criterion (AIC).

The BIC was developed by Gideon E. Schwarz, who gave a Bayesian argument for adopting it. It is closely related to the Akaike information criterion (AIC). In fact, Akaike was so impressed with Schwarz's Bayesian formalism that he developed his own Bayesian formalism, now often referred to as the ABIC for "a Bayesian Information Criterion" or more casually "Akaike's Bayesian Information Criterion".

The BIC is formally defined as

$$BIC = k \ln(n) - 2 \ln(\hat{L})$$

Kolmogorov- Smirnov Goodness of Fit.

The Kolmogorov – Simonov test is a non-parametric test that is used to decide if a sample comes from a population with specific distribution. The K-S test statistic itself does not depend on the underlying cumulative distribution function being tested. It is only applied to continuous distribution and it tends to be more sensitive near the center of the distribution than at the tails.

Test statistic.

$$D = \max_{1 \leq i \leq N} (F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i))$$

Where F is the theoretical cumulative distribution of the distribution being tested which must be a continuous distribution and it must be fully specified (i.e. the location, scale, and shape parameters cannot be estimated from the data).

Kurtosis measures the flatness and peakness of the distribution. For any series to be normally distributed, it must have kurtosis of 3 i.e. Mesokurtic. When kurtosis is greater than 3, it is leptokurtic and when it is less than 3, it is platykurtic

Decision rule: the hypothesis regarding the distributional form is rejected if the test statistic, D ,

is greater than the critical value obtained from the table.

Results

Table 1 shows the summary statistics used in this study. The result indicated that standard deviation of 159.9283 is higher than the general mean of 130.654. The minimum value for the Nigeria GDP is 4.196092 and the maximum value is 568.4989. Since the sample mean is within the maximum and minimum value, the series are good for further analysis. Skewness and kurtosis represent the nature of departure from normality. Skewness measure Positive or negative skewness indicate asymmetry. Skewness measures asymmetry in a data set and is also the degree of distortion from the symmetrical bell curve or the normal distribution. For any series to be normally distributed, it must be symmetrical around the mean, that is, skewness of 0. Skewness of 1.310998 implies that the distribution is positively skewed and it will have a long tail on the right side of the normal distribution.

Kurtosis of 0.2851814 is platykurtic which implies flatness of the distribution curve. This is normally distributed since the p-value ($9.752e^{-05}$) is less than 0.05.

Table 1: Descriptive Analysis of the Nigeria Economic Growth Rate

Descriptive	Value
Mean	130.654
Median	54.24682
Standard Deviation	159.9283
Kurtosis	0.2851814
Skewness	1.310998
Minimum	4.196092
Maximum	568.4989

Table 2: Goodness of fit for the selected distributions

Distributions	Parameters	Values	Kolmogorov-Simnov	P-Values
Normal	Mean	130.6540	0.28835	$6.536e^{-05}$
	Stand. Dev	158.5899		
Weibull	Shape	0.778586	0.13529	0.2028
	Scale	112.286886		
Gamma	Shape	0.6999451	0.15733	0.09178
	Rate	0.0053565		
Cauchy	Location	43.43752	0.23368	0.0023
	Scale	35.41136		
Pareto	Shape	2.035731	0.1007	0.5433
	Scale	154.496136		
Log-Normal	Mean Log	4.009378	0.78151	$6.66e^{16}$
	Std.D. Log	1.452		

The *goodness of fit* test is used to test if sample data fits a distribution from a certain population

Table 2 shows the values of the selected distributions parameters that are used in this study with the Kolmogorov-Smirnov test showing also that the p-value of the data set at 5% level of significance. The values of P- Values determine those that are good fit for the economic growth rate of Nigeria. We conclude that Weibull, Gamma and Pareto distributions are good because the P-Values are greater than 0.05. Using the P-P plots to justify the above claim.

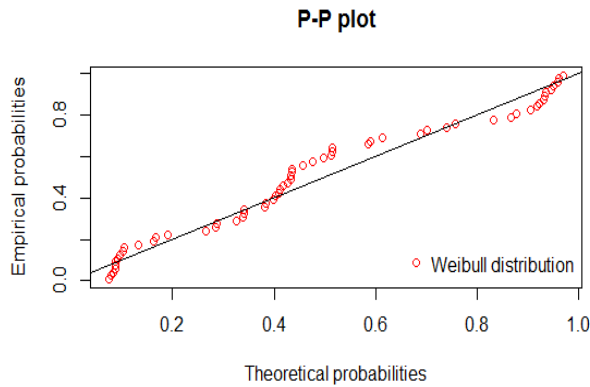
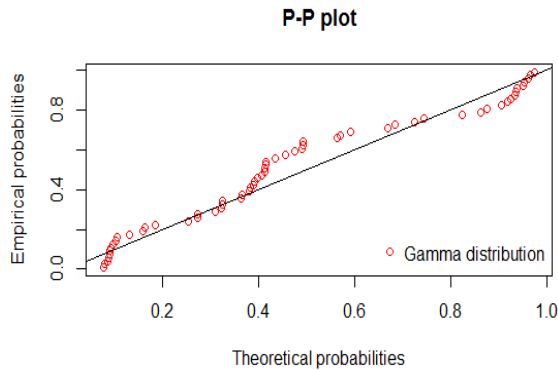


Figure 1: P-P plot of Weibull distribution

A



B

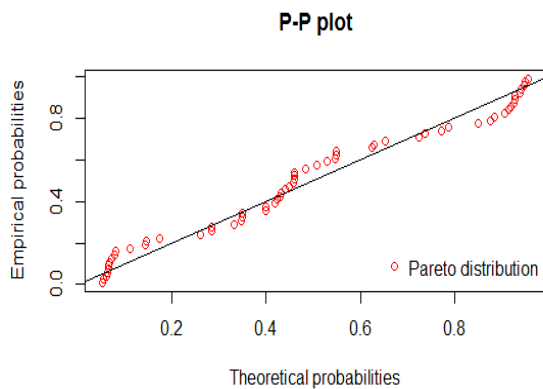


Figure 2 A and B: P-P plot of Gamma and Pareto distribution respectively

Model selection.

Three probability distributions out of the six that were considered in the study, namely, Weibull, Gamma and Pareto distributions is a good fit to the data of economic growth rate of Nigeria. The selection of the best fit model is based on log likelihood, Akaike information Criteria (AIC) and Bayesian Information Criteria (BIC).

Table 3: The log likelihood, AIC, and BIC

Probability distribution	Loglikelihood	AIC	BIC
Weibull	-348.8322	701.6643	705.853
Gamma	-349.4484	702.8968	707.0855
Pareto	-349.2121	702.4243	706.613

From table 3, the Weibull distribution is the best fit model for the data of economic growth rate of Nigeria since it has the highest loglikelihood of -348.8322 and it has the lowest AIC and BIC of 701.6643 and 705.853 respectively.

Computed Probabilities from the Best Fit Distribution.

In order to forecast the performance of the economic growth rate in Nigeria, the best fit probability distribution (Weibull) was used to compute the probabilities of GDP within a certain bound, therefore:

$P(X < 5) = 0.08486632$: This shows that the probability of obtaining an increase in the value of economic growth rate in less than 5 years in Nigeria is 0.0846632 or 8% is very low.

$P(X < 10) = 0.1411275$: This suggests that the probability of gaining an increase in the value of Economic Growth rate in less than 10 years in Nigeria is 0.1411275 or 14% is low.

$P(X > 10) = 0.8588723$: This also implies that the probability of obtaining an increase in the value of

economic growth rate above 10 years in Nigeria is 0.8588723 or 85%, it is high.

Discussion

The Kolmogorov-Smirnov goodness of fit was used to check the six probabilities distribution on how best they fit the yearly economic growth. From the analysis three probability distribution which includes the Weibull distribution, Gamma distribution and Pareto distribution with p-value of 0.2028, 0.09178, 0.5433 respectively which are greater than 0.05 level of significance. Other probability distribution, namely; Normal distribution, Cauchy distribution and the Log-normal distribution did not fit the data.

The probability distribution model that best fit the yearly economic growth rate of Nigeria was selected using the Akaike information criteria and the Bayes information criteria. Between the three distributions that best fits the data set, the Weibull distribution was selected as the best fit probability distribution because it's AIC and BIC are the lowest among the distributions that fits the data.

Probability forecast was done in demand to know the performance of the economic growth rate in Nigeria using the Weibull distribution which best fit probability distribution. It was used to compute the probabilities of GDP within a certain bound.

This shows that the probability of obtaining an increase in the value of economic growth rate in less than 5 years in Nigeria is 8% which is highly low. Also, the probability of obtaining an increase in the value of Economic Growth rate in less than 10 years in Nigeria is 14% which is also low. Lastly, the probability of obtaining an increase in the value of economic growth rate above 10 years in Nigeria is 85% which is high. Meaning that before we can have a meaningful increment in our economic growth in Nigeria is after ten years of this study.

Conclusions

We conclude that Weibull distribution is the best probability distribution to model Nigeria Economic Growth rate. Also, increase in the Gross Domestic Product (GDP) will lead to increase in Economic Growth Rate and vice versa.

We recommend that researchers can now make use of Weibull distribution as an alternative to model Nigeria economic growth rate, make probability forecast and also to know its nature and behaviour.

Conflict of interest

Authors hereby declare that there is no conflicts of interest or other interests that may influence the manuscript. Authors also declare that there is no source of funding for the work undertaken.

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