



An Improved Estimation of Population Mean Using Auxiliary Attribute in Stratified Sampling

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ABSTRACT

Stratification improves the efficiency when the variance between strata is much larger than the variances within strata. An exponential estimator for stratified random sampling was proposed. Data sets on the amount of Apple production taken at three different occasions in Turkey in 1999 were used to validate the proposed estimator. The data sets were stratified by regions of Turkey and from each region; we randomly selected the samples by using the Neyman allocation method. Expressions for MSE of the proposed estimators were derived, up to first order of approximation. The suggested estimator families' bias and MSE equations were computed, the MSE for the proposed estimators is unbiased and efficient. The results of the theoretical comparison and numerical analysis shows that the suggested estimator outperformed the existing estimator. Moreover, the derived exponential estimator performed better than existing estimators for estimating population mean in stratified random sampling with an auxiliary attribute.

Keywords: Auxiliary attribute; Mean square errors (MSE); Percentage Relative Efficiency (PRE); Efficiency; Stratified random sampling.

Introduction

For the exponential liaison between the research variable and the auxiliary attribute, exponential estimators can be used (Sanaullah *et al.*, 2014). In Simple Random Sampling, Bahl and Tuteja (1991) presented the exponential estimator. The exponential estimators in the two-phase sampling were provided by Singh and Vishwakarma (2007), Noor-ul-Amin and Hanif (2012). Koyuncu (2013) proposed a family of exponential estimators for calculating the population mean based on two

auxiliary attributes' information. In the stratified two-phase sampling strategy, Sanaullah *et al.* (2014) introduced the exponential estimators. In simple random sampling, Singh *et al.* (2007) obtained certain exponential ratio-type estimators for the population mean using known values of particular population parameter(s). Shabbir and Gupta (2017) provided a more efficient estimator than a number of competing estimators, including Sanaullah *et al.*, family of estimators (2014) in both simple random and stratified random samplings.

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For estimating the finite population mean, Haq and Shabbir (2013) proposed two better estimator families. In simple random sampling, Lone and Tailor (2017) introduced a new method for estimating population variance. Cekim and Kadilar (2018) enhanced two classes of estimators for estimating the population mean in stratified random sampling in their general form. Under simple random sampling, Zaman and Kadilar (2019) introduced a family of exponential estimators that takes into account auxiliary attributes. In contrast to previous research, the goal of this paper is to present an exponential estimator for estimating the population mean of the studied variable using information about the proportion of the population with specified characteristics in stratified random sampling.

Materials and Methods

In stratified random sampling, the standard combined ratio estimator using a single auxiliary attribute is as follows:

$$t_{rc} = \frac{y_{st}}{p_{st}}$$

where P is the population proportion of the auxiliary attribute and

$$y_{st} = \sum_{k=1}^i \omega_h y_h$$

$$p_{st} = \sum_{k=1}^i \omega_h p_h$$

Where l is the number of stratum, $\omega_h = \frac{N_h}{N}$ is stratum weight, N is the number of units in population, N_h is the number of units in the population stratum, h , y_h is the sample mean of the study variable in the stratum h and p_h is the sample proportion of the auxiliary attribute in the stratum h .

The variance of the combined ratio estimator

$$V(t_{rc}) = \bar{Y}^2 \sum_{h=1}^m \omega_h^2 \mathcal{G}_h (C_{yh}^2 - 2C_{yph} + C_{ph}^2),$$

Where $\mathcal{G}_h = \frac{1 - (\frac{n_h}{N_h})}{n_h}, n_h$ is the number of units in the sample stratum h , C_{yh} is the population coefficient of variation of the auxiliary attribute in the stratum h , and C_{yph} is the population coefficient of variation between the auxiliary attribute and the study variable in the stratum h .

Sharma and Singh (2013) introduced the following exponential estimator:

$$t_d = \bar{y}_{st} \exp\left(\frac{P - p_{st}}{P + p_{st}}\right),$$

The MSE of the estimator given above is

$$MSE(t_d) \cong \sum_{h=1}^m \omega_h^2 \mathcal{G}_h \bar{Y}^2 (\lambda_i^2 C_{ph}^2 - 2\lambda_i C_{yph} + C_{yh}^2),$$

Where $\lambda_1 = \frac{1}{2}$

Tolga(2019) proposed the general class of exponential estimators as follows:

$$t_{stZKl} = \bar{y}_{st} \exp\left[\frac{(kP_{st} + l) - (kp_{st} + l)}{(kP_{st} + l) + (kp_{st} + l)}\right],$$

The bias and MSE equations for the proposed family of estimators, to first order of approximation, are given by

$$B_{(t_{stZKl})} = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h \bar{Y} (\lambda_i^2 C_{ph}^2 - \lambda_i C_{yph}) = \bar{Y} (\lambda_i^2 \beta - \lambda_i \theta);$$

$$MSE_{(t_{stZKl})} = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h \bar{Y}^2 (\lambda_i^2 C_{ph}^2 - 2\lambda_i C_{yph} + C_{yh}^2) = \bar{Y}^2 (\lambda_i^2 \beta - 2\lambda_i \theta + \alpha);$$

where

$$\lambda_1 = \frac{1}{2}; \lambda_2 = \frac{P}{2(P + \beta_2(\phi))}; \lambda_3 = \frac{P}{2(P + C_p)};$$

$$\lambda_4 = \frac{P}{2(P + \rho_{pb})}; \lambda_5 = \frac{\beta_2(\phi)P}{2(\beta_2(\phi)P + C_p)};$$

$$\lambda_6 = \frac{C_p P}{2(C_p P + \beta_2(\phi))}; \lambda_7 = \frac{C_p P}{2(C_p P + \rho_{pb})};$$

$$\lambda_8 = \frac{\rho_{pb} P}{2(\rho_{pb} P + C_p)}; \lambda_9 = \frac{\beta_2(\phi)P}{2(\beta_2(\phi)P + \rho_{pb})};$$

$$\lambda_{10} = \frac{\rho_{pb} P}{2(\rho_{pb} P + \beta_2(\phi))}$$

The general class of exponential estimators Tolga and Kadilar(2020), motivated by Koyuncu and Kadilar(2009) was improved as follows;

$$\delta_i = \kappa \bar{y}_{st} \exp \left[\frac{(kP_{st} + l) - (kp_{st} + l)}{(kP_{st} + l) + (kp_{st} + l)} \right],$$

Where k is a suitable constant to be determined later. Here,

$$\lambda_i = \frac{kP}{2(kP + l)}, i = 1, 2, \dots, 10.$$

Taking the expectation on both sides of the equation, the bias of the estimator given by

$$B(\delta) = \kappa \bar{Y} \sum_{h=1}^m \omega_h^2 \mathcal{G}_h (\lambda_i^2 C_{ph}^2 - \lambda_i C_{yph}) + \bar{Y} (\kappa - 1)$$

Where $\alpha = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h C_{yh}^2$, $\beta = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h C_{ph}^2$,
 $\theta = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h C_{yph}$,

The minimum $MSE(\delta)$ is computed for the optimal value of k,

which is

$$\kappa_{opti} = \frac{2\lambda_i^2 \beta - 2\lambda_i \theta + 2}{6\lambda_i^2 \beta - 8\lambda_i \theta + 2\alpha + 2}; i = 1, 2, \dots, 10.$$

The minimum MSE equations of these estimators were obtained by

$$MSE_{min}(\delta_i) = \bar{Y}^2 [\kappa_{opti}^2 \alpha + \lambda_i^2 (3\kappa_{opti}^2 - 2\kappa_{opti}) \beta - 2\lambda_i (2\kappa_{opti}^2 - \kappa_{opti}) \theta + (\kappa_{opti} - 2)^2];$$

$$i = 1, 2, \dots, 10.$$

Proposed Exponential Estimators

Motivated by Tolga and Kadilar(2019) and Tolga(2020), a general class of exponential estimators were proposed:

$$t_{oj} = \bar{y}_{st} \left[\alpha \exp \left(\frac{(KP_{st} + L) - (Kp_{st} + L)}{(KP_{st} + L) + (Kp_{st} + L)} \right) + (1 - \alpha) \exp \left(\frac{(Kp_{st} + L) - (KP_{st} + L)}{(Kp_{st} + L) + (KP_{st} + L)} \right) \right]$$

To obtain the bias and MSE equations of the estimators, let

$$\bar{y}_{st} = \bar{Y} (1 + e_0), p_{st} = P (1 + e_0) a \quad \text{Such that}$$

$$E(e_0) = E(e_1) = 0; E(e_0^2) = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h C_{yh}^2 = \alpha;$$

$$E(e_1^2) = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h C_{ph}^2 = \beta \quad \text{and}$$

$$E(e_0 e_1^2) = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h C_{yph}^2 = \theta \quad \text{and}$$

$$E(e_0 e_1^2) = \sum_{h=1}^m \omega_h^2 \mathcal{G}_h C_{yph}^2 = \theta.$$

$$\text{let } \bar{y}_{st} = \bar{Y} (1 + \Omega_y), p_{st} = P (1 + \Omega_p), E(\Omega_y) = E(\Omega_p) = 0$$

$$E(\Omega_y^2) = \sum_{h=1}^l \Pi_h^2 \lambda_h C_{yh}^2, E(\Omega_p^2) = \sum_{h=1}^q \Pi_h^2 \lambda_h C_{ph}^2, E(\Omega_y - \Omega_p) = \sum_{h=1}^q \Pi_h^2 \lambda_h C_{yph}$$

$$t_{oj} = \bar{Y} (1 + \Omega_y) \left\{ \begin{aligned} & \alpha \exp \left(\frac{(KP + L) - (KP(1 + \Omega_p) + L)}{(KP + L) + (KP(1 + \Omega_p) + L)} \right) + \\ & (1 - \alpha) \exp \left(\frac{(KP(1 + \Omega_p) + L) - (KP + L)}{(KP(1 + \Omega_p) + L)} \right) \end{aligned} \right\}$$

$$= \bar{Y} (1 + \Omega_y) \left\{ \begin{aligned} & \alpha \exp \left(\frac{-KP\Omega_p}{2(KP + L) + KP\Omega_p} \right) + \\ & (1 - \alpha) \exp \left(\frac{KP\Omega_p}{2(KP + L) + KP\Omega_p} \right) \end{aligned} \right\}$$

$$= \bar{Y} (1 + \Omega_y) \left\{ \begin{aligned} & \alpha \exp \left(-\psi_i \Omega_p (1 + \psi_i \Omega_p)^{-1} \right) \\ & + (1 - \alpha) \exp \left(\psi_i \Omega_p (1 + \psi_i \Omega_p)^{-1} \right) \end{aligned} \right\}$$

$$t_{oj} = \bar{Y} (1 + \Omega_y) \left\{ \begin{array}{l} \alpha \exp[-\psi_i \Omega_p + \psi^2_i \Omega^2_p] \\ + (1 - \alpha) \exp[\psi_i \Omega_p - \psi^2_i \Omega^2_p] \end{array} \right\} \frac{\partial MSE(t_{oj})}{\partial \alpha} = \bar{Y}^2 [(8\alpha - 4)\psi_i B - 4\psi_i C] = 0$$

$$(8\alpha - 4)\psi_i B = 4\psi_i C$$

$$= \bar{Y} + \bar{Y} \left\{ \begin{array}{l} \Omega_y + \alpha \left(-\psi_i \Omega_p + \psi^2_i \Omega^2_p + \frac{\psi^2_i \Omega^2_p}{2} \right) + \\ (1 - \alpha) \left(\psi_i \Omega_p - \psi^2_i \Omega^2_p + \frac{\psi^2_i \Omega^2_p}{2} \right) - 2\alpha \psi_i \Omega_y \Omega_p + \psi_i \Omega_y \Omega_p \end{array} \right\}$$

$$\alpha = \frac{C + \psi_i B}{2\psi_i B}$$

$$\alpha = \frac{C}{2\psi_i B} + \frac{1}{2}$$

$$t_{oj} - \bar{Y} = \bar{Y} \left\{ \begin{array}{l} \Omega_y + \alpha \left(-\psi_i \Omega_p + \psi^2_i \Omega^2_p + \frac{\psi^2_i \Omega^2_p}{2} \right) + \\ (1 - \alpha) \left(\psi_i \Omega_p - \psi^2_i \Omega^2_p + \frac{\psi^2_i \Omega^2_p}{2} \right) \\ - 2\alpha \psi_i \Omega_y \Omega_p + \psi_i \Omega_y \Omega_p \end{array} \right\}$$

$$MSE(t_{oj})_{\min} = \bar{Y}^2 \left[A + \frac{(C + \psi_i B)^2}{B} - 2(C + \psi_i B)\psi_i C - 2\frac{C^2}{B} \right]$$

Efficiency Comparison

$$MSE(t_{oj})_{\min} - Var(y_{st}) < 0, \quad i = 1, 2, \dots, 10.$$

$$\bar{Y}^2 \left[A + \frac{(C + \psi_i B)^2}{B} - 2(C + \psi_i B)\psi_i C - 2\frac{C^2}{B} \right] - \bar{Y}^2 A < 0$$

$$\psi_i^2 B + 2\psi_i C - 2\psi_i C^2 - 2\psi_i^2 BC - \frac{C^2}{B} < 0$$

$$MSE(t_{oj})_{\min} - MSE(t_{td}) < 0,$$

$$i = 1, 2, \dots, 10.$$

$$\bar{Y}^2 \left[A + \frac{(C + \psi_i B)^2}{B} - 2(C + \psi_i B)\psi_i C - 2\frac{C^2}{B} \right] - \bar{Y}^2 \left[\frac{1}{4}B - C + A \right] < 0$$

$$\psi_i^2 B + 2\psi_i C - 2\psi_i C^2 - 2\psi_i^2 BC - \frac{C^2}{B} - \frac{1}{4}B + C < 0$$

$$MSE(t_{oj})_{\min} - MSE(t_{trc}) < 0,$$

$$i = 1, 2, \dots, 10.$$

$$\bar{Y}^2 \left[A + \frac{(C + \psi_i B)^2}{B} - 2(C + \psi_i B)\psi_i C - 2\frac{C^2}{B} \right] - \bar{Y}^2 [B - 2C + A] < 0$$

$$\psi_i^2 B + 2\psi_i C - 2\psi_i C^2 - 2\psi_i^2 BC - \frac{C^2}{B} - B + 2C < 0$$

$$Bias(t_{oj}) = E(t_{oj} - \bar{Y})$$

$$Bias(t_{oj}) = \bar{Y} \left[\begin{array}{l} \alpha \left(\frac{3}{2}\psi_i \sum \Pi_k^2 \lambda_k C_{pk}^2 \right) - (1 - \alpha) \frac{1}{2}\psi_i \sum \Pi_k^2 \lambda_k C_{pk}^2 \\ - 2\alpha \psi_i \sum \Pi_k^2 \lambda_k C_{yph} + \psi_i \sum \Pi_k^2 \lambda_k C_{yph} \end{array} \right]$$

$$= \bar{Y} \left[\left(2\alpha - \frac{1}{2} \right) \psi_i^2 \sum \Pi_k^2 \lambda_k C_{pk}^2 - (2\alpha - 1)\psi_i \sum \Pi_k^2 \lambda_k C_{yph} \right]$$

$$MSE(t_{oj}) = E(t_{oj} - \bar{Y})^2$$

$$= E \left[\bar{Y} \left[\Omega_y - \alpha \psi_i \Omega_p + (1 - \alpha) \psi_i \Omega_p \right] \right]^2$$

$$= E \left[\bar{Y}^2 \left[\Omega_y^2 + 4\alpha^2 \psi_i^2 \Omega_p^2 + \psi_i^2 \Omega_p^2 - 4\alpha \psi_i \Omega_p \right] \right]^2$$

$$= \bar{Y}^2 \sum \Pi_k^2 \lambda_k \left[C_{yh}^2 + (4\alpha^2 - 4\alpha + 1)\psi_i^2 \Omega_{ph}^2 - 2(2\alpha - 1)\psi_i C_{yph} \right]$$

Let $A = \sum \Pi_k^2 \lambda_k C_{yh}^2$, $B = \sum \Pi_k^2 \lambda_k C_{ph}^2$, $C = \sum \Pi_k^2 \lambda_k C_{yph}$

$$MSE(t_{oj}) = \bar{Y}^2 \left[A + (4\alpha^2 - 4\alpha + 1)\psi_i^2 B - 2(2\alpha - 1)\psi_i C \right]$$

$$\text{iv. } MSE(t_{oj})_{\min} - MSE(t_{zki}) < 0, \\ i = 1, 2, \dots, 10. \text{ if}$$

$$\text{Page | 44 } \bar{Y}^2 \left[A + \frac{(C + \psi_i B)^2}{B} - 2(C + \psi_i B)\psi_i C - 2\frac{C^2}{B} \right] - \bar{Y}^2 [\psi_i B - 2\psi_i C + A] < 0$$

$$4\psi_i^2 C + 2\psi_i C^2 - 2\psi_i B C - \frac{C^2}{B} < 0$$

$$\text{v. } MSE(t_{oj})_{\min} - MSE(t_{\bar{a}}) < 0, \quad i = 1, 2, \dots, 10. \text{ if}$$

$$\bar{Y}^2 \left[A + \frac{(C + \psi_i B)^2}{B} - 2(C + \psi_i B)\psi_i C - 2\frac{C^2}{B} \right] -$$

$$\bar{Y}^2 [\kappa_{opt}^2 A + \psi_i^2 (3\kappa_{opt}^2 - 2\kappa_{opt})B - 2\psi_i (2\kappa_{opt}^2 - \kappa_{opt})C + (\kappa_{opt} - 1)^2] < 0$$

$$A(1 - \kappa_{opt}^2) - \psi_i^2 B [2C + (3\kappa_{opt}^2 - 2\kappa_{opt})] + 2\psi_i C [2 - C - (2\kappa_{opt}^2 - \kappa_{opt})]$$

$$+ \frac{C^2}{B} + (\kappa_{opt} - 1)^2 < 0$$

Empirical Study

Study variable is apple production amount in 1999; auxiliary attribute is the number of apple trees more than 15000 in 1999. (Source: Institute of Statistics, Republic of Turkey). We have stratified the data by regions of Turkey (as 1: Marmara 2: Aegean 3: Mediterranean 4: Central Anatolia 5: Black Sea 6: East and Southeast Anatolia) and from each stratum; we have randomly selected the samples whose sizes are computed by using the Neyman allocation method.

Table 1: The first data set of the stratified samples of regions of Turkey

$N = 854$	$N_1 = 106$	$N_2 = 106$	$N_3 = 94$	$N_4 = 171$	$N_5 = 204$	$N_6 = 173$
$n = 200$	$n_1 = 13$	$n_2 = 24$	$n_3 = 55$	$n_4 = 95$	$n_5 = 10$	$n_6 = 3$
$S_{y1} = 6425.0$	$S_{y2} = 11551$	$S_{y3} = 29907$	$S_{y4} = 28643$	$S_{y5} = 2389.7$	$S_{y6} = 945.748$	$S_y = 17105.3$
$\bar{Y}_1 = 6425.08$	$\bar{Y}_2 = 2212.59$	$\bar{Y}_3 = 9384.30$	$\bar{Y}_4 = 5588.01$	$\bar{Y}_5 = 966.956$	$\bar{Y}_6 = 404.399$	$\bar{Y} = 2930.12$
$P_1 = 0.33962$	$P_2 = 0.34900$	$P_3 = 0.53192$	$P_4 = 0.53801$	$P_5 = 0.44118$	$P_6 = 0.18497$	$P = 0.39461$
$C_{y1} = 4.1808$	$C_{y2} = 5.2208$	$C_{y3} = 3.1869$	$C_{y4} = 5.1258$	$C_{y5} = 2.4714$	$C_{y6} = 2.3386$	$C_y = 5.83788$
$C_{p1} = 1.4010$	$C_{p2} = 1.3720$	$C_{p3} = 0.9437$	$C_{p4} = 0.9292$	$C_{p5} = 1.1282$	$C_{p6} = 2.1052$	$C_p = 1.23932$
$\rho_{yp1} = 0.294$	$\rho_{yp2} = 0.240$	$\rho_{yp3} = 0.287$	$\rho_{yp4} = 0.173$	$\rho_{yp5} = 0.346$	$\rho_{yp6} = 0.6142$	$\rho_{yp} = 0.1979$
$C_{yp1} = 1.725$	$C_{yp2} = 1.720$	$C_{yp3} = 0.863$	$C_{yp4} = 0.826$	$C_{yp5} = 0.965$	$C_{yp6} = 3.0240$	$P = 0.39461$
$\hat{\delta}_1 = 0.067$	$\hat{\delta}_2 = 0.032$	$\hat{\delta}_3 = 0.008$	$\hat{\delta}_4 = 0.005$	$\hat{\delta}_5 = 0.095$	$\hat{\delta}_6 = 0.328$	$\beta_2(\phi) = -1.8$
$\omega_1^2 = 0.015$	$\omega_2^2 = 0.015$	$\omega_3^2 = 0.012$	$\omega_4^2 = 0.040$	$\omega_5^2 = 0.057$	$\omega_6^2 = 0.041$	
$\alpha = 0.14422$	$\beta = 0.06969$	$\theta = 0.04877$	$\lambda_1 = 0.5$	$\lambda_1 = -0.138$	$\lambda_3 = 0.12076$	$\lambda_4 = 0.33296$
$\lambda_5 = -0.686$	$\lambda_6 = -0.184$	$\lambda_7 = 0.3559$	$\lambda_8 = 0.02965$	$\lambda_9 = 0.6906$	$\lambda_{10} = -0.0224$	

Table 2: The second data set of the stratified samples of regions of Turkey

$N = 854$	$N_1 = 106$	$N_2 = 106$	$N_3 = 94$	$N_4 = 171$	$N_5 = 204$	$N_6 = 173$
$n = 200$	$n_1 = 13$	$n_2 = 24$	$n_3 = 55$	$n_4 = 95$	$n_5 = 10$	$n_6 = 3$
$S_{y1} = 6425.1$	$S_{y2} = 11551.1$	$S_{y3} = 29907.4$	$S_{y4} = 28643$	$S_{y5} = 2389.1$	$S_{y6} = 945.7$	$\xi = 7.8748279$
$\bar{Y}_1 = 1536.77$	$\bar{Y}_2 = 2212.59$	$\bar{Y}_3 = 9384.309$	$\bar{Y}_4 = 5588.01$	$\bar{Y}_5 = 966.951$	$\bar{Y}_6 = 404.39$	$\psi = 0.0030109$
$P_1 = 0.2452$	$P_2 = 0.29241$	$P_3 = 0.46808$	$P_4 = 0.48538$	$P_5 = 0.3627$	$P_6 = 0.1156$	$R = 8780.0021$
$S_{p1} = 0.432$	$S_{p1} = 0.4571$	$S_{p3} = 0.5017$	$S_{p4} = 0.5013$	$S_{p5} = 0.481$	$S_{p6} = 0.320$	$R_{pro1SD} = 1623.$
$\beta_{21}(\varphi) = 0.4$	$\beta_{22}(\varphi) = 1.11$	$\beta_{23}(\varphi) = -2.0$	$\beta_{24}(\varphi) = 2.0$	$\beta_{25}(\varphi) = 1.6$	$\beta_{26}(\varphi) = 3.9$	$R_{pro2SK} = 1519.$
$C_{p1} = 1.725$	$C_{p2} = 1.5631$	$C_{p3} = 1.0718$	$C_{p4} = 1.0328$	$C_{p5} = 1.328$	$C_{p6} = 2.773$	$R_{pro3US1} = 1547$
$\rho_{yp1} = 0.358$	$\rho_{yp2} = 0.265$	$\rho_{yp3} = 0.3249$	$\rho_{yp4} = 0.191$	$\rho_{yp5} = 0.389$	$\rho_{yp6} = 0.671$	$R_{pro3US2} = 1435$
$\mathcal{G}_1 = 0.067$	$\mathcal{G}_2 = 0.032$	$\mathcal{G}_3 = 0.008$	$\mathcal{G}_4 = 0.005$	$\mathcal{G}_5 = 0.095$	$\mathcal{G}_6 = 0.328$	$\xi^* = 0.9438$
$\omega_1^2 = 0.015$	$\omega_2^2 = 0.015$	$\omega_3^2 = 0.012$	$\omega_4^2 = 0.040$	$\omega_5^2 = 0.057$	$\omega_6^2 = 0.041$	
$S_{yp1} = 996.5$	$S_{yp2} = 1404.1$	$S_{yp3} = 48674.$	$S_{yp4} = 2743.1$	$S_{yp5} = 449.1$	$S_{yp6} = 204.1$	$R_{pro4ST} = 4099$
$P_{SD} = 1.804$	$P_{SK} = 1.9287$	$P_{US1} = 1.8928$	$P_{US2} = 2.041$	$P_{ST} = 0.714$	$P = 0.33371$	$\bar{Y} = 2930.127$

Table 3: The third data set of the stratified samples of regions of Turkey

$N = 854$	$N_1 = 106$	$N_2 = 106$	$N_3 = 94$	$N_4 = 171$	$N_5 = 204$	$N_6 = 173$
$n = 200$	$n_1 = 13$	$n_2 = 24$	$n_3 = 55$	$n_4 = 95$	$n_5 = 10$	$n_6 = 3$
$S_{y1} = 6425.0$	$S_{y2} = 11551.1$	$S_{y3} = 29907.1$	$S_{y4} = 28643$	$S_{y5} = 2389.1$	$S_{y6} = 945.7$	$\xi = 8.2408358$
$\bar{Y}_1 = 1536.77$	$\bar{Y}_2 = 2212.59$	$\bar{Y}_3 = 9384.30$	$\bar{Y}_4 = 5588.01$	$\bar{Y}_5 = 966.951$	$\bar{Y}_6 = 404.39$	$\psi = 0.0023394$
$P_1 = 0.22641$	$P_2 = 0.25471$	$P_3 = 0.40426$	$P_4 = 0.39766$	$P_5 = 0.2156$	$P_6 = 0.0867$	$R = 11584.763$
$S_{p1} = 0.4205$	$S_{p1} = 0.4377$	$S_{p3} = 0.4933$	$S_{p4} = 0.4908$	$S_{p5} = 0.412$	$S_{p6} = 0.282$	$R_{pro1SD} = 1335.$
$\beta_{21}(\varphi) = 0.2$	$\beta_{22}(\varphi) = 0.7$	$\beta_{23}(\varphi) = -1.$	$\beta_{24}(\varphi) = 1.8.$	$\beta_{25}(\varphi) = 0.1$	$\beta_{26}(\varphi) = 6.1$	$R_{pro2SK} = 1512$
$C_{p1} = 1.8572$	$C_{p2} = 1.7186$	$C_{p3} = 1.2204$	$C_{p4} = 1.2343$	$C_{p5} = 1.911$	$C_{p6} = 3.254$	$R_{pro3US1} = 1358$
$\rho_{yp1} = 0.380$	$\rho_{yp2} = 0.292$	$\rho_{yp3} = 0.366$	$\rho_{yp4} = 0.226$	$\rho_{yp5} = 0.519$	$\rho_{yp6} = 0.711$	$R_{pro3US2} = 1395$
$\mathcal{G}_1 = 0.067$	$\mathcal{G}_2 = 0.032$	$\mathcal{G}_3 = 0.008$	$\mathcal{G}_4 = 0.005$	$\mathcal{G}_5 = 0.095$	$\mathcal{G}_6 = 0.328$	$\xi^* = 0.9438$
$\omega_1^2 = 0.015$	$\omega_2^2 = 0.015$	$\omega_3^2 = 0.012$	$\omega_4^2 = 0.040$	$\omega_5^2 = 0.057$	$\omega_6^2 = 0.041$	
$S_{yp1} = 1029.1$	$S_{yp2} = 1481.1$	$S_{yp3} = 5408.4$	$S_{yp4} = 3190.1$	$S_{yp5} = 511.5$	$S_{yp6} = 190.9$	$R_{pro4ST} = 4238$
$P_{SD} = 2.1941$	$P_{SK} = 1.9374$	$P_{US1} = 2.1571$	$P_{US2} = 2.099$	$P_{ST} = 0.691$	$P = 0.25291$	$\bar{Y} = 2930.127$

Table 4: MSEs and PREs of Proposed and Existing Estimators using Data 1

ESTIMATORS	MSE	PRE
y_{st}	1240285	100
t_{rc}	1001021	123.9021
t_d	970903.1	127.7455
t_{zk2}	1368026	90.66243
t_{zk3}	1147792	108.0584
t_{zk4}	1027583	120.6992
t_{zk5}	2098816	59.09453
t_{zk6}	1414859	87.66143
t_{zk7}	1017811	121.8581
t_{zk8}	1215958	102.0007
t_{zk9}	947059.7	130.9617
t_{zk10}	1259415	98.48107
d_1	882942.2	140.4719
d_2	1163244	106.623
d_3	1021047	121.4719
d_4	931359.3	133.1694
d_5	1496245	82.89319
d_6	1190539	104.1785
d_7	923476.7	134.3061
d_8	1067688	116.1656
d_9	853643.5	145.2931
d_{10}	1096133	113.151
t_{oj}	660589.1	187.7545

Table 5: MSEs and PREs of Proposed and Existing Estimators using Data 2

ESTIMATORS	MSE	PRE
y_{st}	1240285	100
t_{rc}	1414036	87.71246
t_d	1074157	115.466
t_{zk2}	1375966	90.13928
t_{zk3}	1153815	107.4943
t_{zk4}	1073371	115.5505
t_{zk5}	2293724	54.07301
t_{zk6}	1428849	86.80309
t_{zk7}	1070131	115.9003
t_{zk8}	1216321	101.9703
t_{zk9}	1144050	108.4119
t_{zk10}	1259623	98.46478
d_1	946008.9	131.1072
d_2	1167128	106.2682
d_3	1024536	121.0582
d_4	959449.3	129.2706
d_5	1539604	80.55873
d_6	1197106	103.607
d_7	955637.3	129.7862
d_8	1067888	116.1437
d_9	967272.4	128.225
d_{10}	1096245	113.1395
t_{oj}	899947	137.8176

Table 6: MSEs and PREs of Proposed and Existing Estimators using Data 3

ESTIMATORS	MSE	PRE
y_{st}	1240285	100
t_{rc}	1843160	67.29126
t_d	1181438	104.981
t_{zk2}	1384215	89.60208
t_{zk3}	1160073	106.9145
t_{zk4}	1120945	110.6464
t_{zk5}	2496233	49.68628
t_{zk6}	1443386	85.9289
t_{zk7}	1124492	110.2974
t_{zk8}	1216698	101.9387
t_{zk9}	1348723	91.95999
t_{zk10}	1259840	98.44787
d_1	1005182	123.3892
d_2	1171131	105.905
d_3	1028140	120.6339
d_4	987340.7	125.6188
d_5	1574346	78.781
d_6	1203832	103.0281
d_7	987365.3	125.6157
d_8	1068097	116.1211
d_9	1064414	116.5229
d_{10}	1096360	113.1275
t_{oj}	1003302	123.6203

Under stratified random sampling, exponential families of estimators were proposed. The suggested estimator families' bias and MSE equations were computed. The proposed estimator's MSE and PRE was computed, and the proposed estimator had the lowest Mean Square Error(MSE). In comparison to other existing estimators, the Percentage Relative Efficiency(PRE) is higher as it was reported in

Table 4, 5 and 6. As a result, the proposed estimator outperforms the existing estimators.

The findings of theoretical comparisons shows that the suggested estimator families outperform the competing estimators. Also, the numerical analysis presented here corroborates this conclusion.

Conflict of Interest

Authors hereby declare that there is no conflicts of interest or other interest that may influence the manuscript.

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